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Friction Coefficients for Bubbly Two-Phase Flow in Horizontal Pipes

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Friction coefficients determined from an experimental study of the flow of bubbly two-phase mixtures in horizontal pipes are used, together with a homogeneous theory, to establish a method for the prediction of the static pressure drop in this class of flows.

SCOPE

The results of experiments on high velocity flows of bubbly two-phase mixtures, that is, mixtures in which the gas phase is dispersed as bubbles throughout a continuous liquid phase, are presented. A simple mathematical model describing this class of flows has been developed by Huey and Bryant (1967), which depends for its application on the availability of experimentally determined average friction coefficients. Huey and Bryant concluded, on the basis of restricted experimental data for air-water mixtures, that friction coefficients could be correlated satisfactorily by a suitably defined Reynolds number alone. They also suggested that, as a first approximation, friction coefficients for single phase flows at the same equivalent Reynolds number could be adopted. The effect of Mach number on the coefficients was thought to be small.

The purpose of this investigation was to obtain reliable experimental data over a wider range of the flow variables and thereby to confirm or disprove these hypotheses and to develop a method for predicting the static pressure drop. A statistical examination of the results obtained was undertaken to establish working formulae correlating friction coefficients with the Reynolds number Re, the pipe diameter D, and the mixture quality χ which would be applicable more generally in the experimental range. These equations were used to predict the static pressure which could then be compared with the experimentally observed results.

CONCLUSIONS AND SIGNIFICANCE

The experimentally determined average friction coefficients $\overline{C_f}$ are shown to be dependent on the mass flow ratio and the pipe diameter, as well as Reynolds number. Therefore, the suggestion of Huey and Bryant (1967) is not substantiated by the experimental evidence reported.

With the correlation equations presented, the average friction coefficients can be predicted, within the experimental range, with an average error of the order of 2%.

The apparent variation of $\overline{C_f}$ with pipe length is shown to be primarily due to compressibility effects. If a high velocity and low pressure region exists, the void fraction can increase substantially, and large accelerations and high Mach numbers can occur. Under these conditions the homogeneous theory is shown to over-estimate the acceleration component in the pressure drop. This inadequacy is thought to be a consequence of the occurrence of slip between the phases. However, regions of high Mach number are of restricted extent in practical flows, and the validity of the homogeneous theory has therefore been substantiated.

When the equations given for $\overline{C_f}$ are used, together with the homogeneous theory, the average error in the prediction of the pressure drop is ±3% in the experimental range. It is shown, however, that the results can be more generally applied, subject to stated restrictions.

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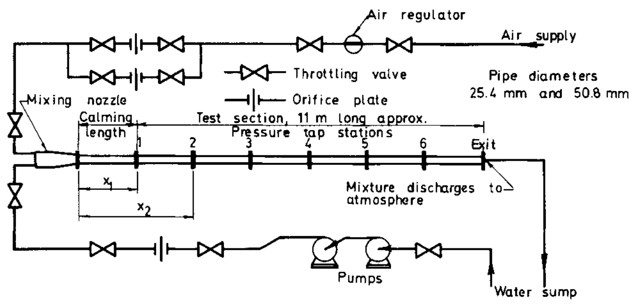

Fig. 1. Schematic flow diagram.

A method is developed for the prediction of the static pressure drop occurring in the flow of bubbly two-phase mixtures in horizontal pipes, which uses mean friction coefficients based on experimental data. Here, the term *bubbly* is used to describe the flow regime where the gas phase is uniformly dispersed as bubbles within a continuous liquid phase. The experiments were conducted with air-water mixtures.

The flow is made amenable to analysis by the assumptions that the mixture is homogeneous and that no slip occurs between the phases. In the method employed the proportion of the static pressure drop required to accelerate the flow is estimated from momentum considerations; that is, from the momentum equation which can be expressed in the form

$$\frac{dp}{dx}\left[1-\frac{\alpha\rho u^2}{p}\right]+4\frac{C_f}{D}\cdot\frac{1}{2}\rho u^2=0 \qquad (1)$$

where $\alpha\rho u^2/p$ is the acceleration term. The remainder is then attributed to the effects of shear stress at the boundaries and is identified with an average friction coefficient defined by

$$\overline{C}_f=\frac{1}{x-x_1}\int_{x_1}^{x}C_f\,dx \qquad (2)$$

This coefficient can be expressed solely in terms of experimentally measured quantities. However, Huey and Bryant (1967) have shown, by a development of the analysis, that an analogy can be drawn with the isothermal flow of an ideal gas. This approach is adopted in the present work because, by introducing a quantity analogous to Mach number and, with this, the concept of a limiting flow condition, the equations describing the flow can be rationalized in the manner usual in gas dynamics. The average friction coefficient can then be found from†

$$\frac{4}{D}\overline{C}_f(x_1-x)=2\frac{\delta^*}{\gamma^{1/2}}\left(\frac{1}{M}-\frac{1}{M_1}\right)$$
$$+2(1-\delta^{*2})\ln\frac{1+\gamma^{1/2}\delta^*M}{1+\gamma^{1/2}\delta^*M_1}-2\delta^{*2}\ln\frac{M_1}{M} \qquad (3)$$

It should be noted that for any initial Mach number and length of pipe there is a limiting value of the friction coefficient above which this equation does not have a solution; this limit is defined by $M = 1/\sqrt{\gamma}$.

It is also convenient to correlate the friction coefficients, so determined, with a suitably defined Reynolds number,† for then, by an analysis of variance, working formulae can be established.

The validity of these formulae and their accuracy are verified by using them to predict the static pressure drop and by comparing these predictions with measured values.

† Parameters M, γ, δ^* and Re are defined in Huey and Bryant (1967).

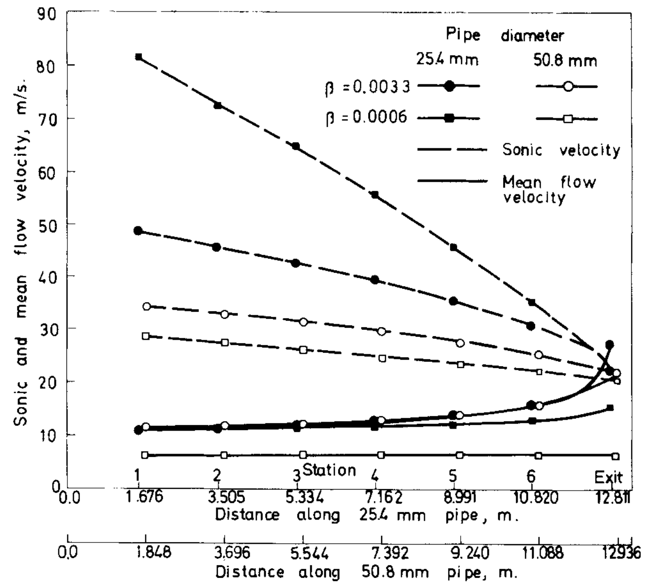

Fig. 2. Variations of sonic and mean flow velocities.

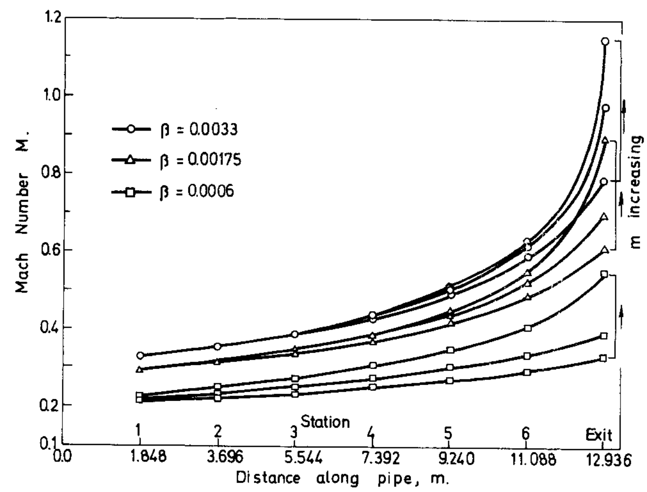

Fig. 3. Variation of Mach number along 50.8 mm diam. pipe.

EXPERIMENTAL EQUIPMENT, MEASUREMENTS AND PROCEDURE

The schematic layout of the experimental equipment is shown in Figure 1.

The static (gauge) pressures recorded varied from 0.7 to 256 kN/m² in the 50.8 mm pipe and from 0.7 to 496 kN/m² in the 25.4 mm pipe. Pressure fluctuations ranging from 2% at 25 cm Hg to 5% at 2.5 cm Hg were noticed near the exit plane in flows with G_l close to 4 880 kg/s m² and with high air concentrations; otherwise the readings were steady and repeatable.

Most of the experimental data were obtained under conditions where the pipe discharged freely into the atmosphere. However, in some tests with the 50.8 mm pipe the exit plane pressure was elevated by an external constraint. Some results were also obtained in a pipe 12.8 m long. The static pressure distribution was measured in more detail near the exit.

The temperature of the mixture was observed to remain constant in all the flows examined.

The range of variables studied was as follows:

Diameter of pipe	25.4 mm	50.8 mm
β	0.0006-0.0033	0.0006-0.0033
α at inlet	0.09-0.42	0.17-0.48
α at the exit plane	0.33-0.73	0.33-0 73
G_l	4,880-10,700 kg/s m²	4,880-7,800 kg/s m²
Re	1.4×10^5-3.0×10^5	2.4×10^5-4.3×10^5

EXPERIMENTAL RESULTS AND THEIR INTERPRETATION

Flow Parameters as Determined from the Homogeneous Model

Flow parameters and properties can be inferred by using a homogeneous flow theory if the static pressure p, the rates of flow of water and air m_l and m_g and the temperature of the mixture T are known. The results showed that variations in these parameters were small in the first half of the pipe. However, the flow parameters changed more rapidly near the exit, and particularly so at higher velocities (Figures 2 and 3).*

Compressibility effects cannot be ignored in bubbly flows. Flows with $\beta = 0.0006$ underwent small density changes; however, the ratio $(\rho_{\text{inlet}} - \rho_{\text{exit}})/\rho_{\text{exit}}$ increased substantially as β and the mass flow rate of the mixture increased. The effects of compressibility were even more noticeable in the 25.4 mm pipe, for the same values of β, than in the 50.8 mm pipe.

The sonic velocity and Mach number† distributions along the pipe (Figures 2 and 3) demonstrate the differences between isothermal compressible gas flow and two-phase flow at constant temperature. In isothermal gas flow the sonic velocity is constant along the pipe, being a function of temperature only. This is not so in the case of bubbly flow, where the sonic velocity varies (Figure 2), and the rapid increase in Mach number (Figure 3) near the exit is due to two effects: an increase in velocity of the flow and a fall in the sonic velocity. Furthermore, isothermal gas flow can only occur, in practice, at relatively low Mach numbers, whereas bubbly two-phase flow can be accelerated to high Mach numbers without incurring change in temperature, since there is no requirement for heat transfer between the flow and surroundings. The flow as a whole remains essentially adiabatic.** Hence, although there is close similarity between the equations describing the flows (Huey and Bryant, 1967), they are not physically analogous.

Friction and Acceleration Pressure Gradients

The acceleration component $\alpha\rho u^2/p$ in Equation (1) is dominant in the last 1.8 m of pipe where, because of the rapid pressure drop (and low absolute pressure), the specific volume of the gas phase increases sharply. This effect is more pronounced at high mass flow rates with β large.

An analysis of the experimental results showed that the homogeneous model over-estimate the acceleration component. [Under some conditions the calculated values of $\alpha\rho u^2/p$ were larger than unity near the exit plane, which is physically impossible.] In this context it should be noted that the acceleration component is a function of the void fraction and the density of the mixture

$$\left[\frac{\alpha\rho u^2}{p} = \frac{d}{\rho p}\left(\frac{m}{A}\right)^2\right].$$

Therefore, any error in prediction of this component can be attributed to the assumption of no slip between the phases, because slip is the predominant factor affecting α and ρ. (The effect of slip is to reduce α and increase ρ.) The errors are most noticeable at low absolute pressures and in the presence of steep pressure gradients, where the differences in phase density and the rate of change of mixture density are greatest and the accelerations highest.

It must also be noted that the homogeneous theory gives $M > 1/\sqrt{\gamma}$ corresponding to $\alpha\rho u^2/p > 1$.

* Additional results may be found in Kopalinsky (1971).

† Defined in Huey and Bryant (1967).

** In the bubbly mixture the mass and thermal capacity of the liquid are large compared with those of the gas. Thus, when air-water mixtures are expanded, the liquid phase acts as a heat reservoir largely suppressing gas phase temperature changes.

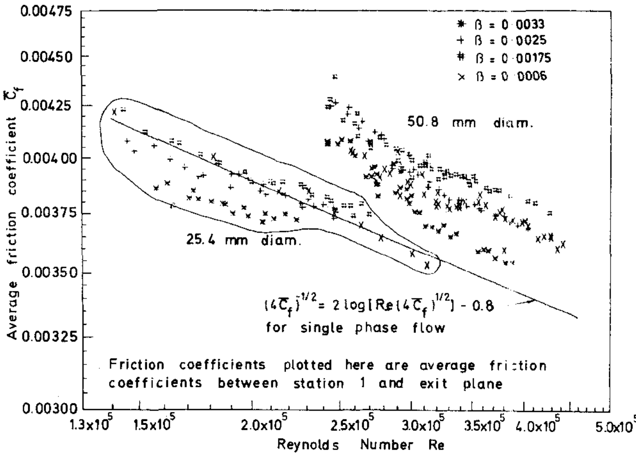

Fig. 4. Average friction coefficient vs. Reynolds number.

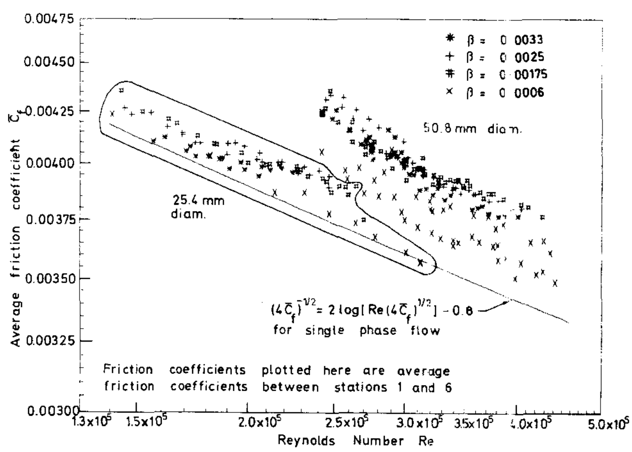

Fig. 5. Average friction coefficient vs. Reynolds number.

Average Friction Coefficients

Friction coefficients averaged over different pipe lengths (between station 1 and successive stations downstream) were computed from Equation (3).

The average friction coefficients for fixed flow conditions varied little with length of pipe. (The maximum difference was found to be approximately 10%.) However, as will be seen later, a small variation in the friction coefficient can have a large effect on the estimated static pressure and, hence, on the predicted flow variables.

The degree of variation in the friction coefficients with pipe length depended upon compressibility effects in the flow. In the 50.8 mm pipe and for flows with β less than 0.0025 (that is, when compressibility effects are not marked) the friction coefficients were nearly constant. However, in flows with higher values of β the average coefficients tended to decrease progressively as pipe sections closer to the exit plane were included in the overall test length, and particularly so when the last section was added. Since compressibility effects were greater in the 25.4 mm pipe, the variations were more pronounced in this case.

The average friction coefficients were also affected by the mass flow rate of the mixture m, the mass flow ratio β and the diameter of the test pipe D. This can be seen from Figures 4 and 5.

FRICTION COEFFICIENT EQUATIONS

An analysis of variance was performed on the data given in Figures 4 and 5. When data for both sizes were combined, the following results were obtained:*

* Other expressions are available in Kopalinsky (1971).

1. Including the exit plane (that is, a region at atmospheric pressure), $\Delta x = 11$ m approximately

$$\overline{C}_f = 0.00516 + 0.278\,\chi - 81.894\,\chi^2 - 0.000869 \ln (Re \times 10^{-5}) - 0.1977 \times 10^{-5} \frac{\Delta x}{D} \qquad (4a)$$

(Maximum errors $\pm$ 4.7% and $\pm$ 3.6% for the 50.8 and 25.4 mm case, respectively.)

2. Excluding the exit plane, $\Delta x = 9.2$ m approximately

$$\overline{C}_f = 0.00504 + 0.374\,\chi - 84.819\,\chi^2 - 0.000894 \ln (Re \times 10^{-5}) - 0.2094 \times 10^{-5} \frac{\Delta x}{D} \qquad (4b)$$

(Maximum error $\pm$ 7.3%.)

It has already been remarked that the very steep pressure gradients which occur in the flow adjacent to the exit plane cause rapid changes in the flow variables. Therefore, while the experimental results do not differentiate between end effects and the influence simply of low absolute pressure in the exit plane, Equation (4a), which includes high Mach number data, should be used in preference to (4b), when $M > 0.7$. With this provision, the equations given can be applied to flows in differing pipe lengths providing the flow conditions (that is, the pressure ranges) are similar to those obtaining in the experiments. The validity of this procedure was confirmed by a satisfactory prediction of $\overline{C}_f$* for flows in the 12.8 m long pipe.

PREDICTION OF STATIC PRESSURE DROP

Computational Procedure

The static pressure at any point along the flow is determined from†

$$p = \frac{p_1 M_1}{M} \qquad (5)$$

where the Mach number can be computed iteratively from Equation (3) by using Equation (4) to estimate the friction coefficient $\overline{C}_f$. The estimated friction coefficients must, however, be compared with the limiting values calculated from Equation (3) with $M = 1/\sqrt{\gamma}$. If the estimated $\overline{C}_f$ is greater than the limiting value, the chosen flow cannot exist, and the initial conditions must be reviewed. In practice this situation will be met only when atmospheric pressure is approached in high Mach number flows.

Comparison of Predicted and Experimental Results

The static pressure drop Δp was calculated for all values of the experimental inlet conditions. In some instances the estimates of $\overline{C}_f$ for pipe lengths containing the exit plane were larger than the limiting value. In these cases, the pressure was predicted on the basis of the limiting value of $\overline{C}_f$. The largest and average errors** occurring in predictions of Δp between station 1 and the exit plane and stations 1 and 6 were 14 and 3%, respectively.

In Figure 6 pressure drops predicted for the 50.8 mm pipe are compared with measured values. It can be seen that the predicted and measured values are in closer agreement in the lower range of overall pressure drops (less than 140 kN/m², corresponding to low exit plane Mach numbers M_E) than in the upper range (corresponding to high M_E). The static pressure is predicted with lower accuracy at high Mach numbers because of the form of Equation (3). Thus, the relationship between the average friction coefficient and the Mach number for given initial conditions exhibits a maximum in the region of $M = 1$, and so in this region relatively small changes in the friction coefficient cause large variations in the Mach number prediction. In cases where the exit plane pressure was raised above atmospheric (that is, $M_E < 0.7$), the agreement between the predicted and experimental pressure drop was found to be good (the average error was about 2%).

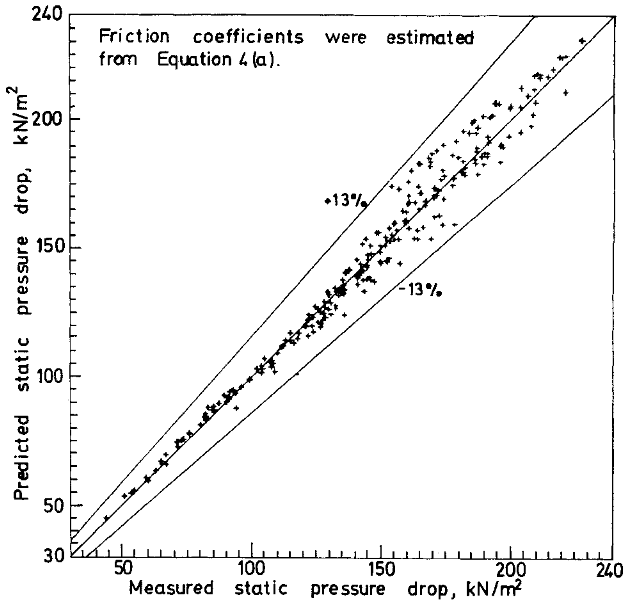

Fig. 6. Comparison of predicted and measured static pressure drops between station 1 and exit. 50.8 mm diam. pipe.

Pressures predicted at different pipe stations are compared with those measured in Figure 7.

The Lockhart-Martinelli method (Lockhart and Martinelli, 1949), which is still considered (DeGance and Atherton, 1970) to be more reliable and consistent than other methods, was used to estimate the pressure drop for the present experimental data for comparison purposes. The correlation with the experimental results was poor: the errors in Δp between station 1 and the exit plane were between 40 and 118% in the 25.4 mm pipe and up to 60% in the 50.8 mm pipe.

CONCLUSIONS

The experimentally determined average friction coefficients $\overline{C}_f$ are shown to be dependent on the mass flow ratio and the pipe diameter as well as on the Reynolds number. Therefore, the suggestion of Huey and Bryant (1967) and Wallis (1969) that average friction coefficients for the liquid phase at the same equivalent Reynolds number can be adopted in the bubbly regime of two-phase flow is not substantiated by the experimental evidence reported here.

The influence of the mass flow ratio is marked at higher values ($\beta > 0.002$) and particularly so at low absolute pressures (that is, close to atmospheric), when the void fraction, and consequently the Mach number, increases substantially. Thus, the variation of $\overline{C}_f$ with pipe length is primarily a consequence of the magnitude of compressibility effects (that is, of the absolute pressure excursion from inlet to outlet) and not simply due to length alone.

The accuracy and validity of the calculation of $\overline{C}_f$ (as well as of the mixture density and other flow parameters)

* The estimated $\overline{C}_f$ were compared with the experimental values, and the error was defined as $(\overline{C}_{f\,est} - \overline{C}_{f\,exp})/\overline{C}_{f\,exp} \times 100\%$.

† Vide a M-δ relationship at constant temperature (Huey and Bryant, 1967).

** Errors are defined by $[(p_1 - p_{prd})/(p_1 - p_{exp}) - 1] \times 100\%$.

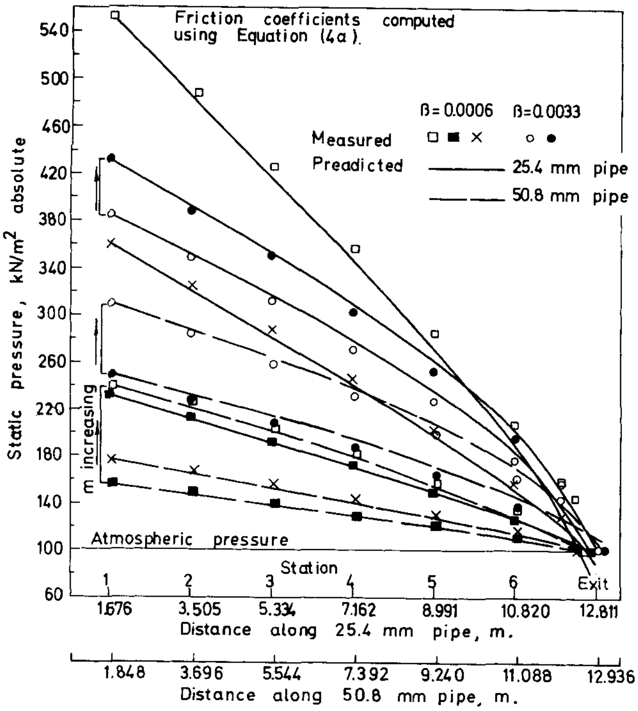

Fig. 7. Comparison of predicted and experimental static pressure distributions.

from the experimentally recorded pressure drop data are also prejudiced by compressibility effects; for when high Mach numbers are achieved large accelerations occurs and deviations from the no slip and homogeneity assumptions, inherent to the analysis, become significant. Specifically, the acceleration component of the pressure drop is then over-estimated by the theory. However, because of the large accelerations occurring, high Mach number flow is necessarily restricted to very limited regions.

When values of C_f estimated from the given regression equations are used (together with the homogeneous theory) to predict the pressure drop, a small error in $\overline{C}_f$ is reflected in a relatively larger error in the prediction of the Mach number (and of the pressure) for $M > 0.8$.

The pressure drop can be predicted satisfactorily for pipe lengths differing from those employed in the experiments. However, when this is done the equations given for $\overline{C}_f$ which exclude exit plane data should not be used when the Mach number exceeds 0.8 approximately, for then these results may predict a friction coefficient which exceeds the limiting value.

When the equations given for $\overline{C}_f$ are applied within the bounds of the observed flows, the average error in the prediction of the static pressure drop is about $\pm$ 3%.

NOTATION

a = apparent sonic velocity, $a^2 = \left(\frac{1+\delta}{\delta}\right)^2 \gamma RT$
A = area
c_l = specific heat of the liquid phase
c_p = specific heat at constant pressure, $c_p = (c_l + \beta c_{pg})/(1+\beta)$
c_v = specific heat at constant volume, $c_v = (c_l + \beta c_{vg})/(1+\beta)$
C_f = local friction coefficient
$\overline{C}_f$ = average friction coefficient, see Equation (2)
D = pipe diameter
G = mass flow rate per unit area of pipe cross section
m = mass rate of flow
M = Mach number, $M = \frac{u}{a}$
p = static pressure
Q = volumetric rate of flow
R = constant in equation of state, $R = \frac{\beta}{1+\beta} R_g$
Re = Reynolds number, $Re = \frac{\rho D u}{\mu_l} = (1+\beta)\, Re_l$
T = absolute temperature
u = mean flow velocity, $u = \frac{Q}{A} = \frac{m_g/\rho_g + m_l/\rho_l}{A}$
x = pipe length

Greek Letters

α = void fraction, $\alpha = \frac{Q_g}{Q_g + Q_l}$
β = mass flow ratio, $\beta = m_g/m_l$
δ = volume flow ratio, $\delta = Q_g/Q_l = \beta\rho_l/\rho_g$
γ = ratio of specific heats for the mixture
Δx = pipe length, $\Delta x = (x - x_1)$
μ = dynamic viscosity
ρ = density, $\rho = \left(\frac{\beta}{1+\beta}\cdot\frac{1}{\rho_g} + \frac{1}{1+\beta}\cdot\frac{1}{\rho_l}\right)^{-1}$
χ = quality of mixture, $\chi = m_g/(m_g + m_l)$

Subscripts

g = gas phase
l = liquid phase
$*$ = critical conditions
exp = experimental values
prd = predicted values
1 = reference station 1

Symbols without the subscripts g and l relate to properties of the mixture

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